

# BRF/albedo inversion constrained by temporal smoothness

P. Lewis<sup>1</sup>, T. Quaife<sup>1</sup>, C. B. Schaaf<sup>2</sup>  
M. Román<sup>2</sup>, Z. Wang<sup>2</sup>, Y. Shuai<sup>2</sup>

1. NCEO and Dept. Geography, UCL, Gower St. London WC1E 6BT, UK  
[plewis@geog.ucl.ac.uk](mailto:plewis@geog.ucl.ac.uk)

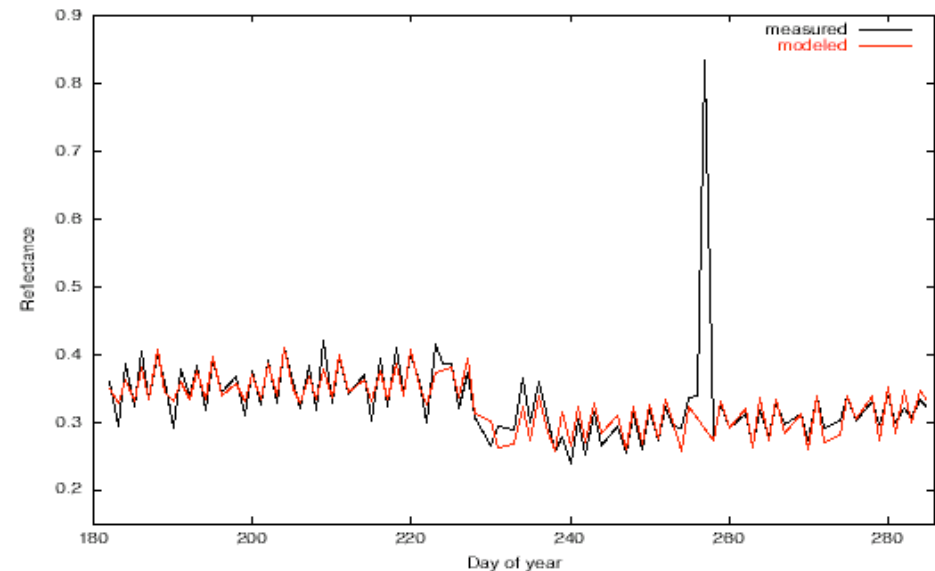
2. Department of Geography and Environment, Boston University, Boston, MA, USA

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Mr. Yu, "Yucheng Experiment Station(YCES), Chinese Academy of Sciences (CAS) "

# Context: Albedo from daily BRF

## ◆ Linear models of BRF

- Angular normalisation
- Signal tracking
- Integrals to estimate albedo
- Operational – e.g. MODIS, Seviri



$$\mathbf{f} = (\mathbf{K}^T \mathbf{C}^{-1} \mathbf{K})^{-1} \mathbf{K}^T \mathbf{C}^{-1} \boldsymbol{\rho}$$

# Twomey smoother

- ◆ Apply Twomey smoother
  - Solve constrained linear problem
  - Lagrange multipliers
- ◆ Closely related to
  - Tikhonov regularisation
  - Kalman smoother
- ◆ Widely used in atmos. RS for profile retrieval

Twomey, Introduction to the Mathematics of Inversion in Remote Sensing and Indirect Measurements. Dover Publications, 1996

Quaife and Lewis, submitted, TGRS, Temporal constraints on linear BRF model parameters

# Twomey smoother

$$\mathbf{f}^* = \begin{pmatrix} \mathbf{K}^{*T} \mathbf{C}^{-1} \mathbf{K}^* + \mathbf{B}^T \mathbf{\Gamma}^2 \mathbf{B} \\ \mathbf{K}^{*T} \mathbf{C}^{-1} \boldsymbol{\rho} + \mathbf{B}^T \mathbf{\Gamma}^2 \mathbf{q} \end{pmatrix}^{-1}.$$

$\mathbf{B}, \mathbf{q}$  specifies required constraint

$$\mathbf{B}\mathbf{f}^* = \mathbf{q}$$

e.g.  $df/dt = 0$  (first order differences)

$$\mathbf{f}^* = \begin{bmatrix} f_{1,1} \\ \vdots \\ f_{1,t} \\ f_{2,1} \\ \vdots \\ f_{n,t} \end{bmatrix}$$

$\mathbf{\Gamma}$  is a weighting operator, with  $\mathbf{\Gamma}^2 = \mathbf{\Gamma}^T \mathbf{\Gamma}$

Note eqn. similar form to Bayesian statement of problem:  
Think of constraint as prior (Yinhong Li et al. 2005 IGARSS)

# Twomey smoother

$$\mathbf{f}^* = \left( \mathbf{K}^{*T} \mathbf{C}^{-1} \mathbf{K}^* + \gamma^2 \mathbf{B}^T \mathbf{B} \right)^{-1} \mathbf{K}^{*T} \mathbf{C}^{-1} \boldsymbol{\rho}$$

e.g.  $df/dt = 0$  (first order differences)

$$\mathbf{K}^* = \begin{bmatrix} K_1(\Omega_1, \Omega'_1) & & & & \\ & \ddots & & & \\ & & K_1(\Omega_m, \Omega'_m) & & \\ K_2(\Omega_1, \Omega'_1) & & & & \\ & \ddots & & & \\ & & K_2(\Omega_m, \Omega'_m) & & \\ \vdots & \vdots & \vdots & & \\ K_n(\Omega_1, \Omega'_1) & & & & \\ & \ddots & & & \\ & & K_n(\Omega_m, \Omega'_m) & & \end{bmatrix}$$

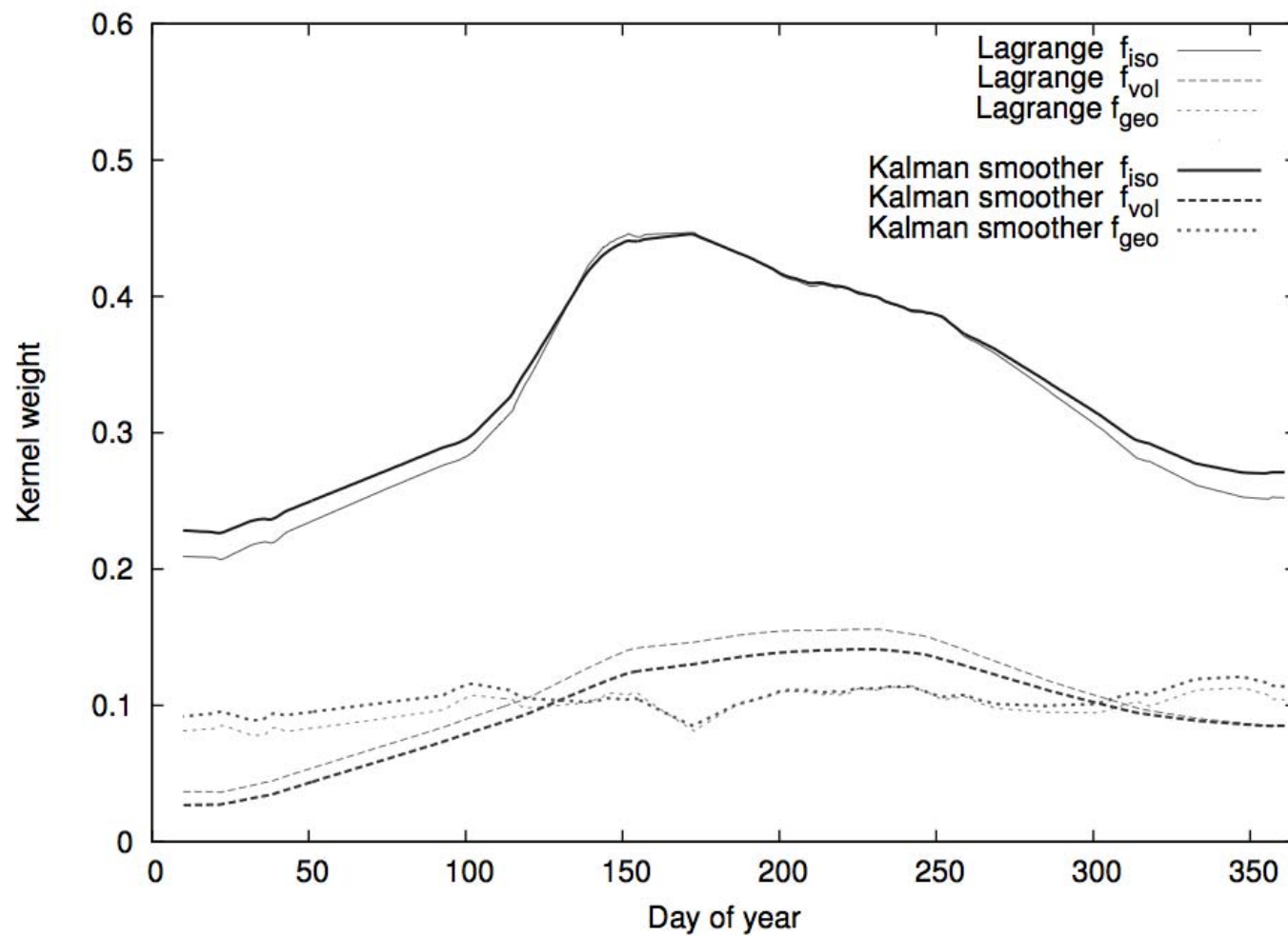
$$\mathbf{B} = \begin{bmatrix} 0 & 0 & & & \\ -1 & 1 & 0 & & \\ 0 & -1 & 1 & 0 & \\ & \ddots & \ddots & \ddots & \ddots \\ & & 0 & -1 & 1 & 0 \\ & & & 0 & -1 & 1 \end{bmatrix}$$

$$\mathbf{f}^* = \begin{bmatrix} f_{1,1} \\ \vdots \\ f_{1,t} \\ f_{2,1} \\ \vdots \\ f_{n,t} \end{bmatrix}$$

$\gamma$ — Lagrange multiplier

confidence in assumed dynamics (e.g. no change)

# Kalman smoother



# issues/options with method

## ◆ what constraints?

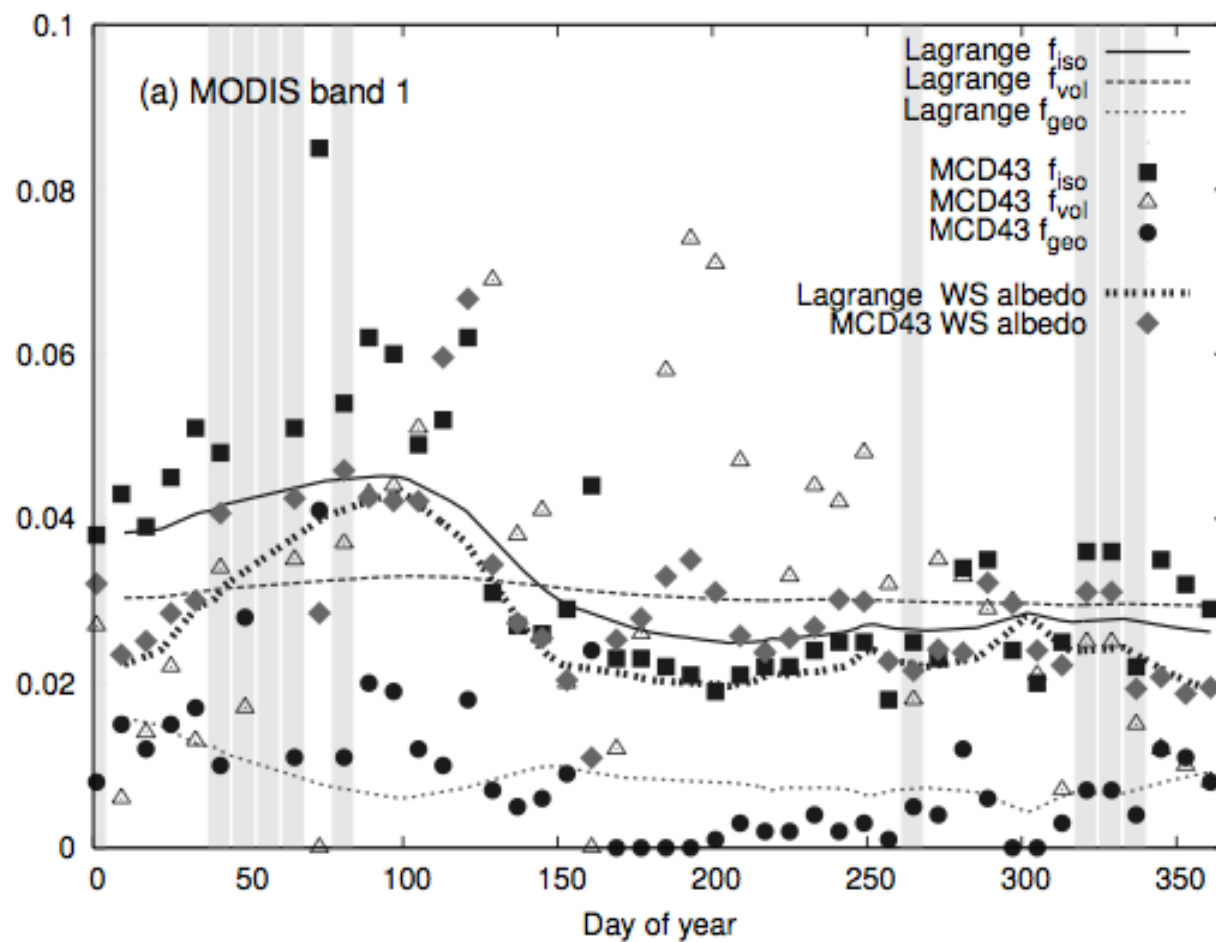
- e.g. 1st Order / 2nd Order

## ◆ degree of smoothing

- $\gamma^2$
- for multiple params?
- Constant over time?

# Spain h17v04

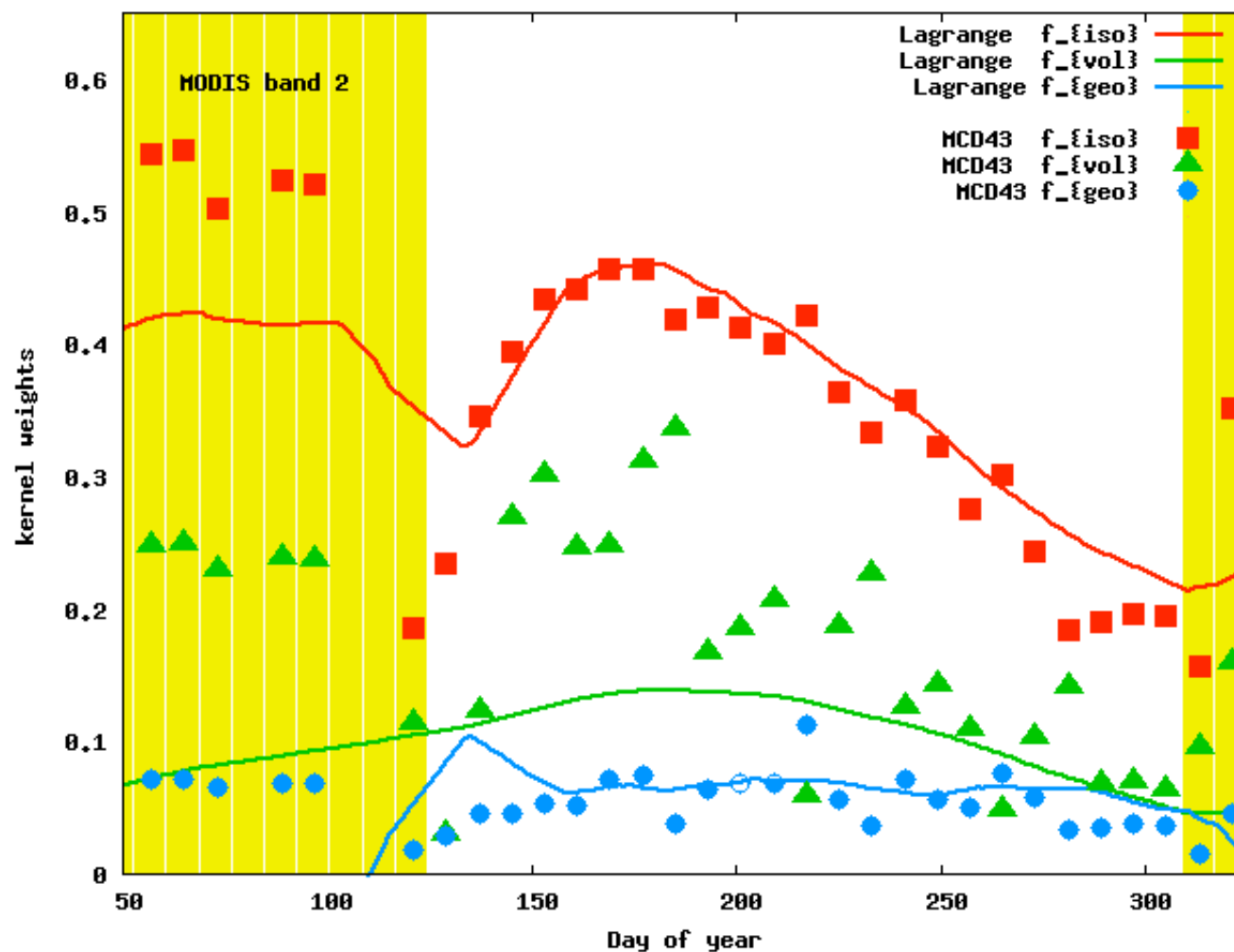
## Deciduous forest



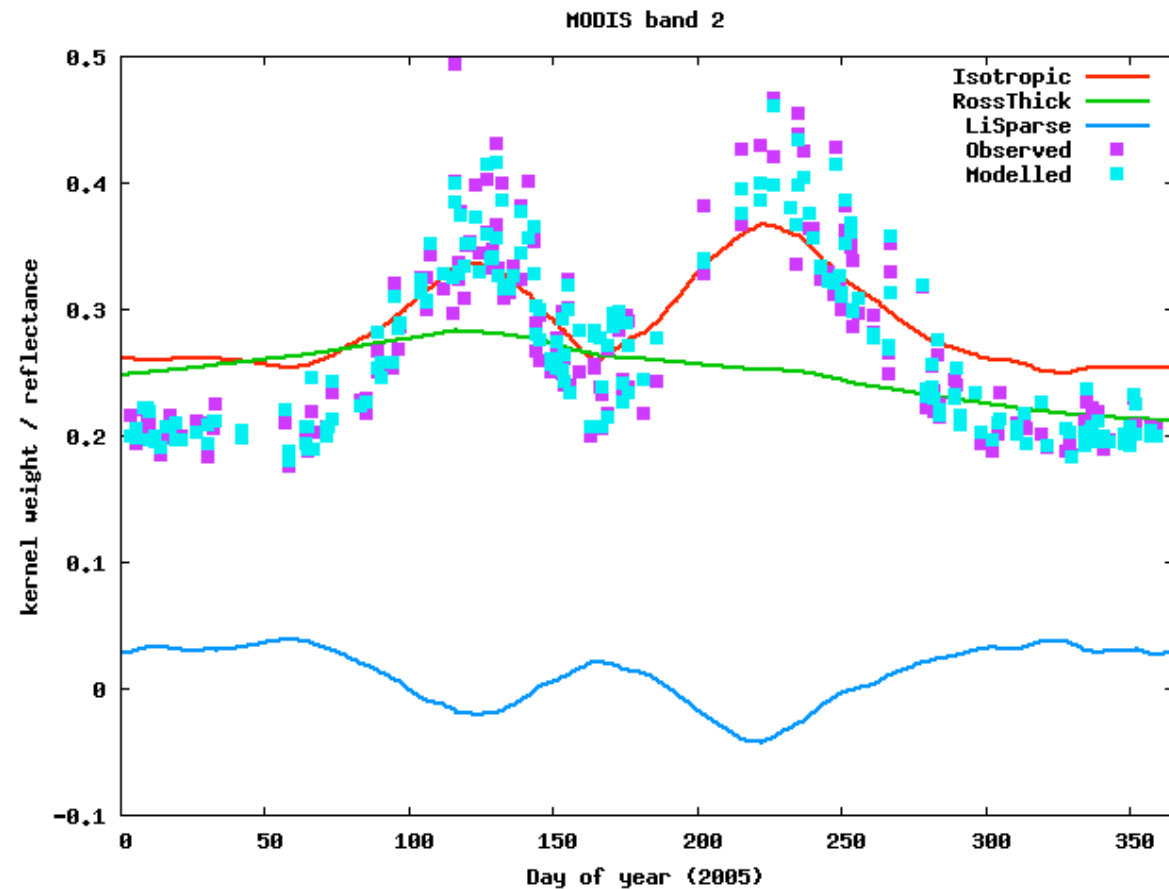


# Siberia h23v03

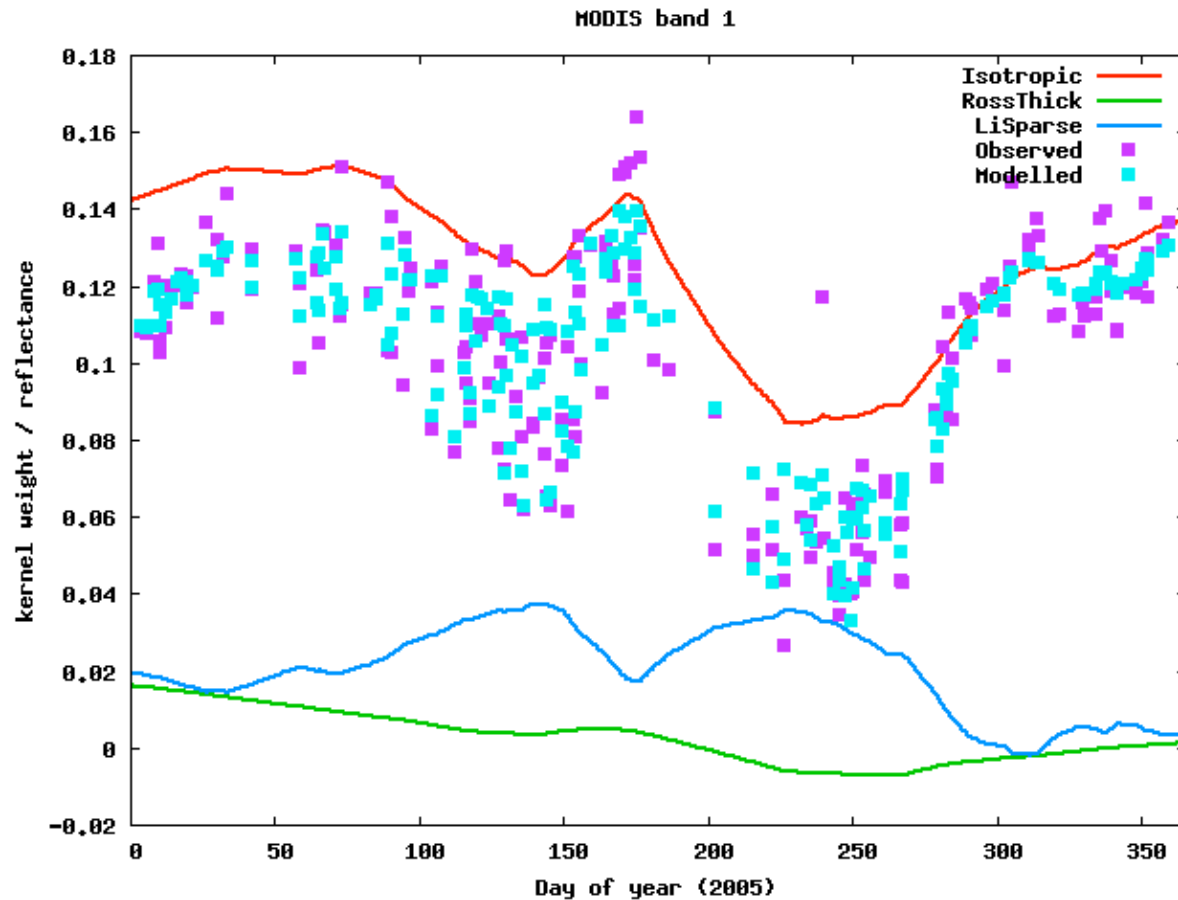
## Deciduous forest



# Agricultural site: Yucheng Experiment Station, Chinese Academy of Sciences h27v05 2004

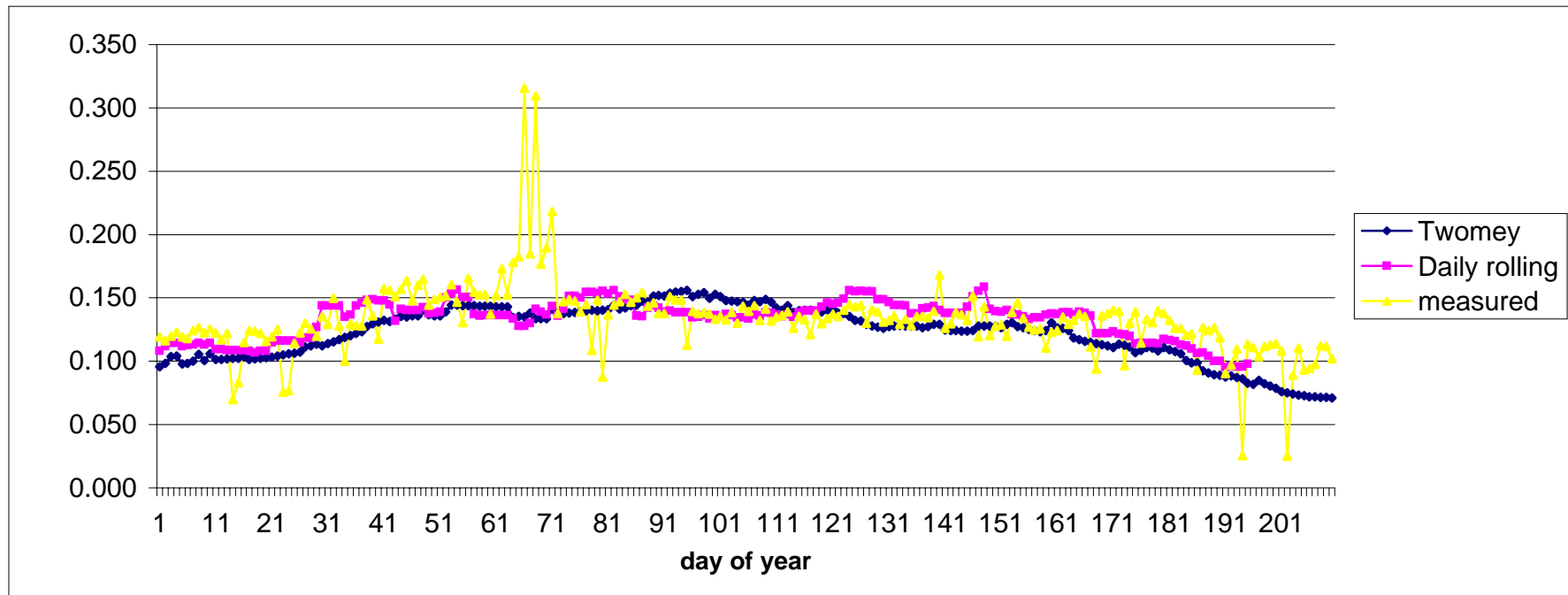


# Agricultural site: Yucheng Experiment Station, Chinese Academy of Sciences h27v05 2004

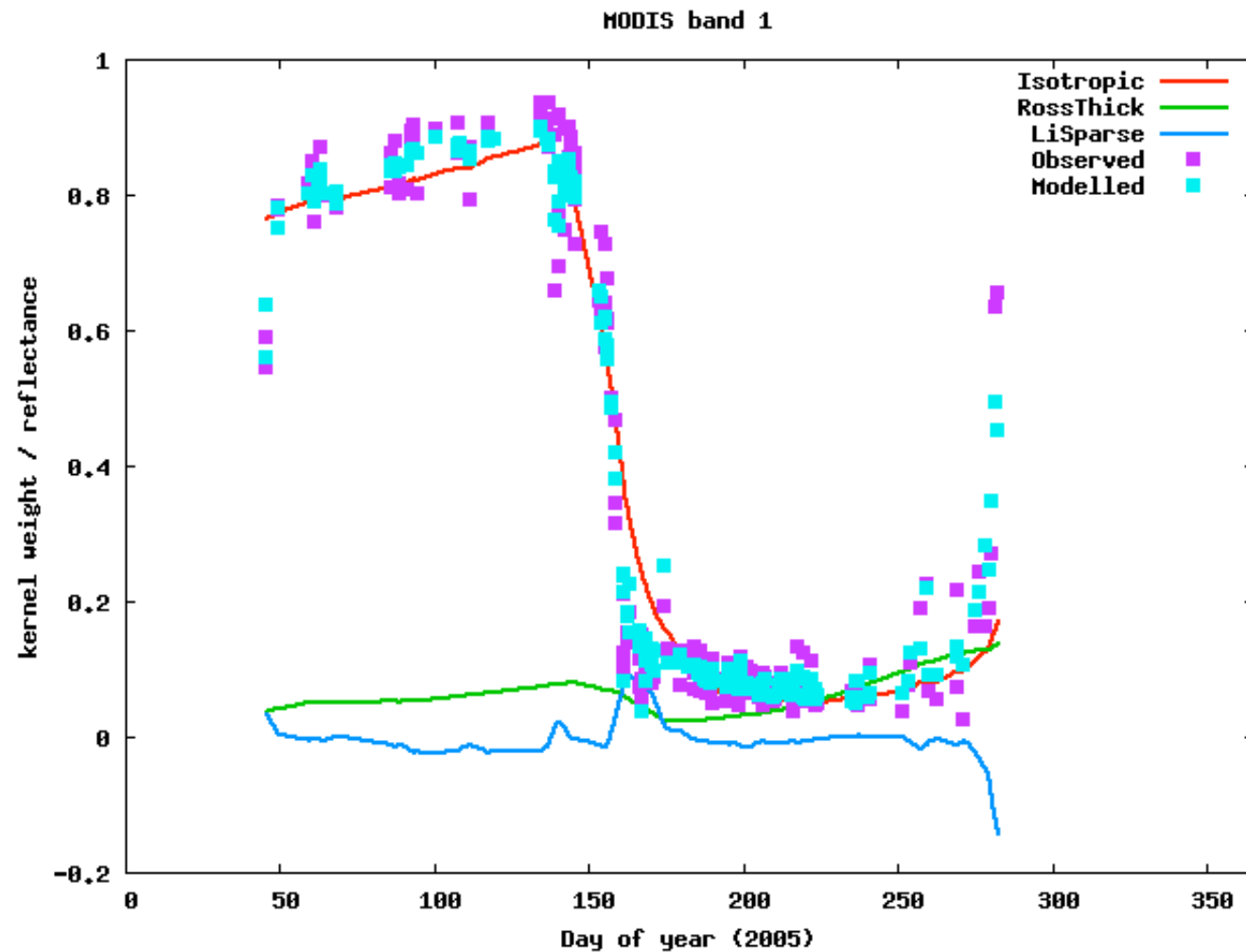


# Agricultural site: Yucheng Experiment Station, Chinese Academy of Sciences h27v05 2006

## ◆ Albedo comparison



# Barrow 2007



# Discussion

- ◆ Constraint-based method
  - Lagrangian
  - Similar in concept to Bayesian priors
  - Related to DA methods
    - ◆ Kalman smoother
    - ◆ E.g. here, 'prior' is zero-order process model

# Discussion

- ◆ Advantages:
  - Framework for ‘knowledge’/expectation
    - ◆ Of state or dynamics of state
  - Same concepts apply spatially
  - Allows for daily model parameters
    - ◆ Can operate in regions of sparse sampling

# Discussion

- ◆ (apparent) Disadvantages:
  - Needs to deal with strong step edges better
    - ◆ Hardly surprising for a smoothing algorithm
    - ◆ Same applies to moving window
  - Need to make choices re what information
    - ◆ E.g. smoothness
  - And how much
    - ◆ Degree of smoothness



# Discussion

- ◆ **BUT** in a sense simply being explicit about this
  - E.g. smoothness imposed by
    - ◆ 16-day windows
    - ◆ Temporal weighting within window
  - Same concepts as in DA
    - ◆ Need data uncertainty and model uncertainty to fully specify

# Where next?

- ◆ Interesting method of constraining albedo through smoothness
  - Esp. for regions of sparse sampling
  - heritage from atmospheric RS.
  - Related to DA methods
- ◆ Examine how to learn from experience of MODIS BRDF/albedo from 2000+
  - E.g. expectations of state or smoothness
- ◆ Less restrictive than current backup algorithm assumptions
  - Although that operates very well, considering ...

# Thanks

◆ Thank you!

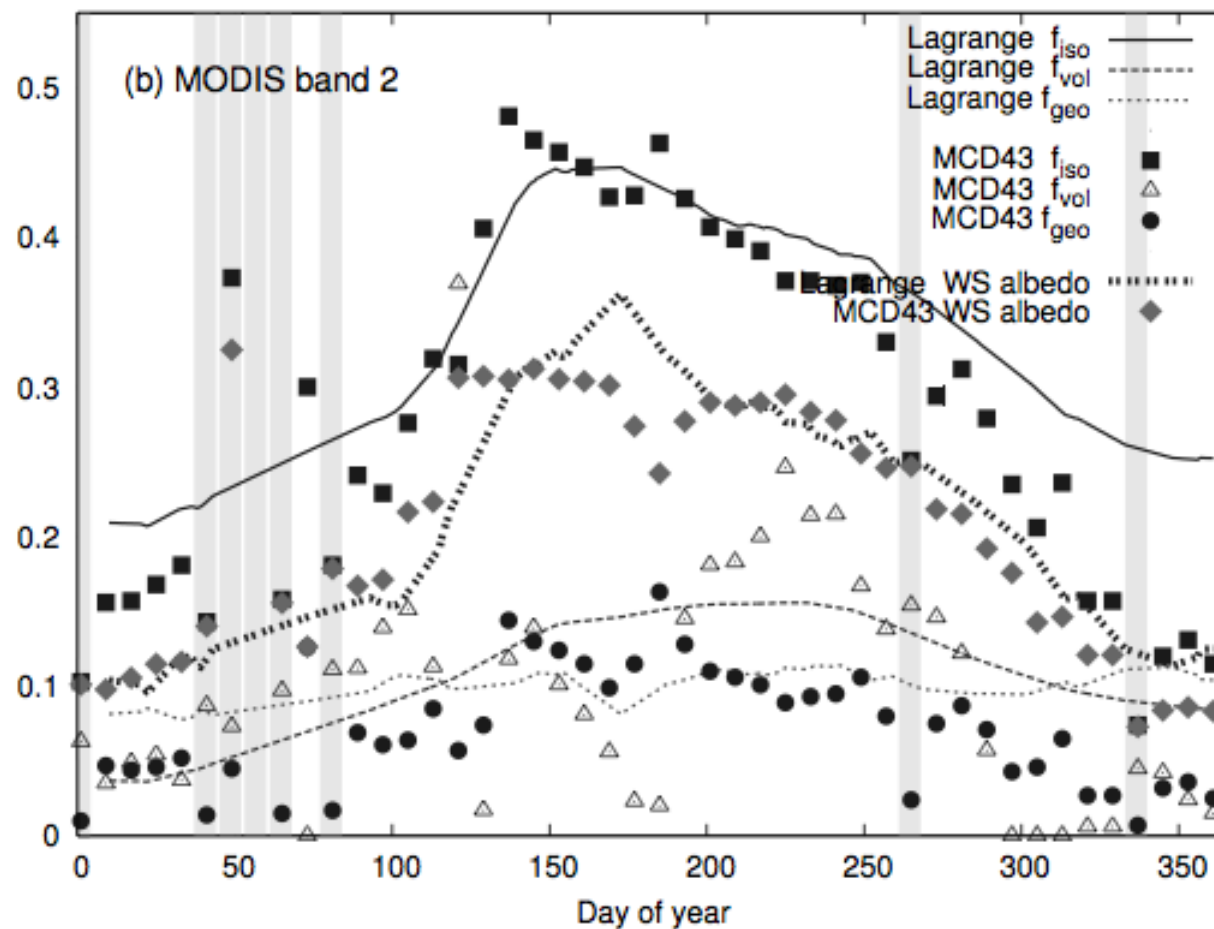
# Spare slides

# results

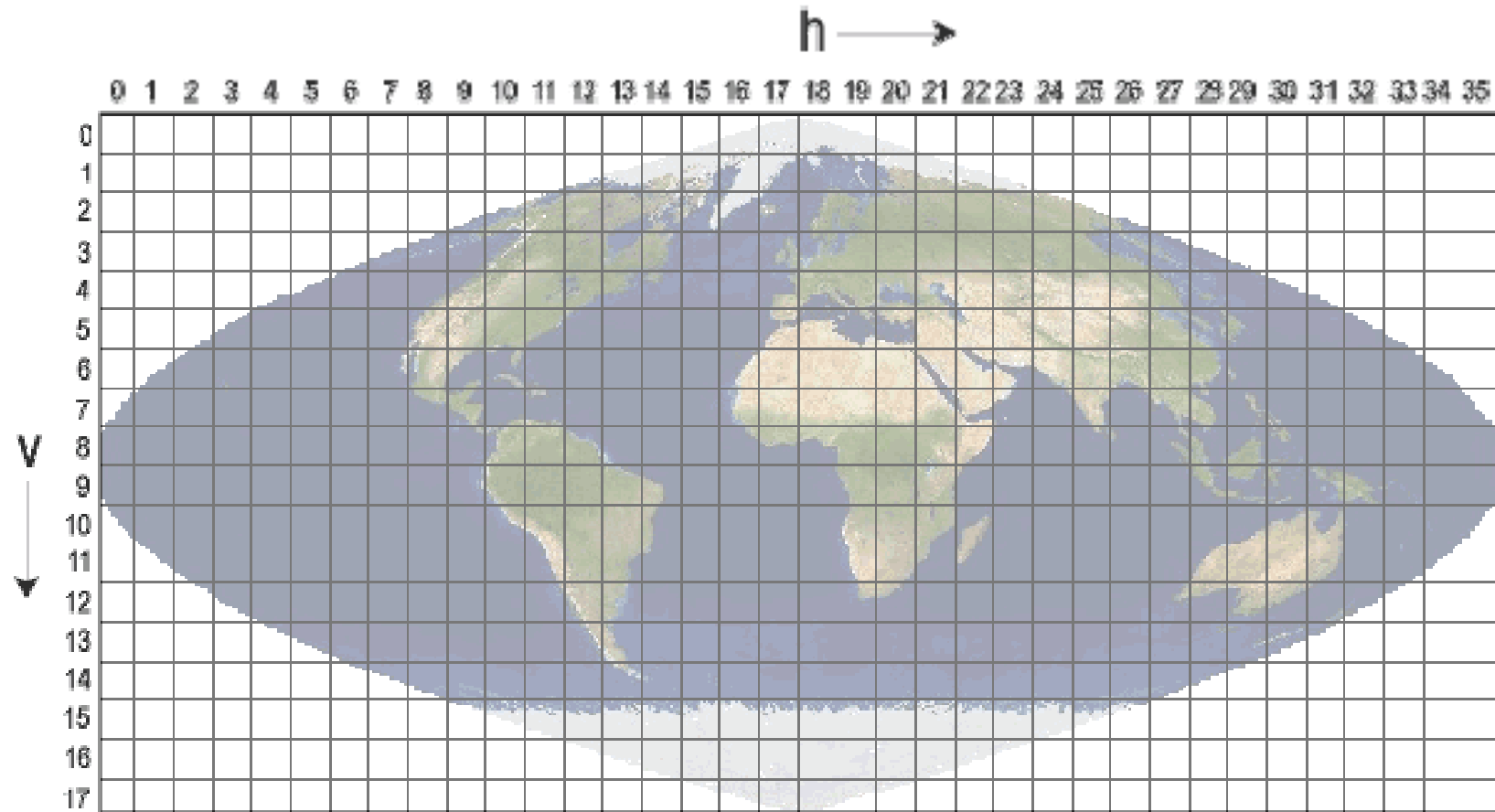
- ◆ Spain/Siberia
  - Comparison of MCD43 and smoother
- ◆ Barrow
- ◆ Yucheng

# Spain h17v04

## Deciduous forest

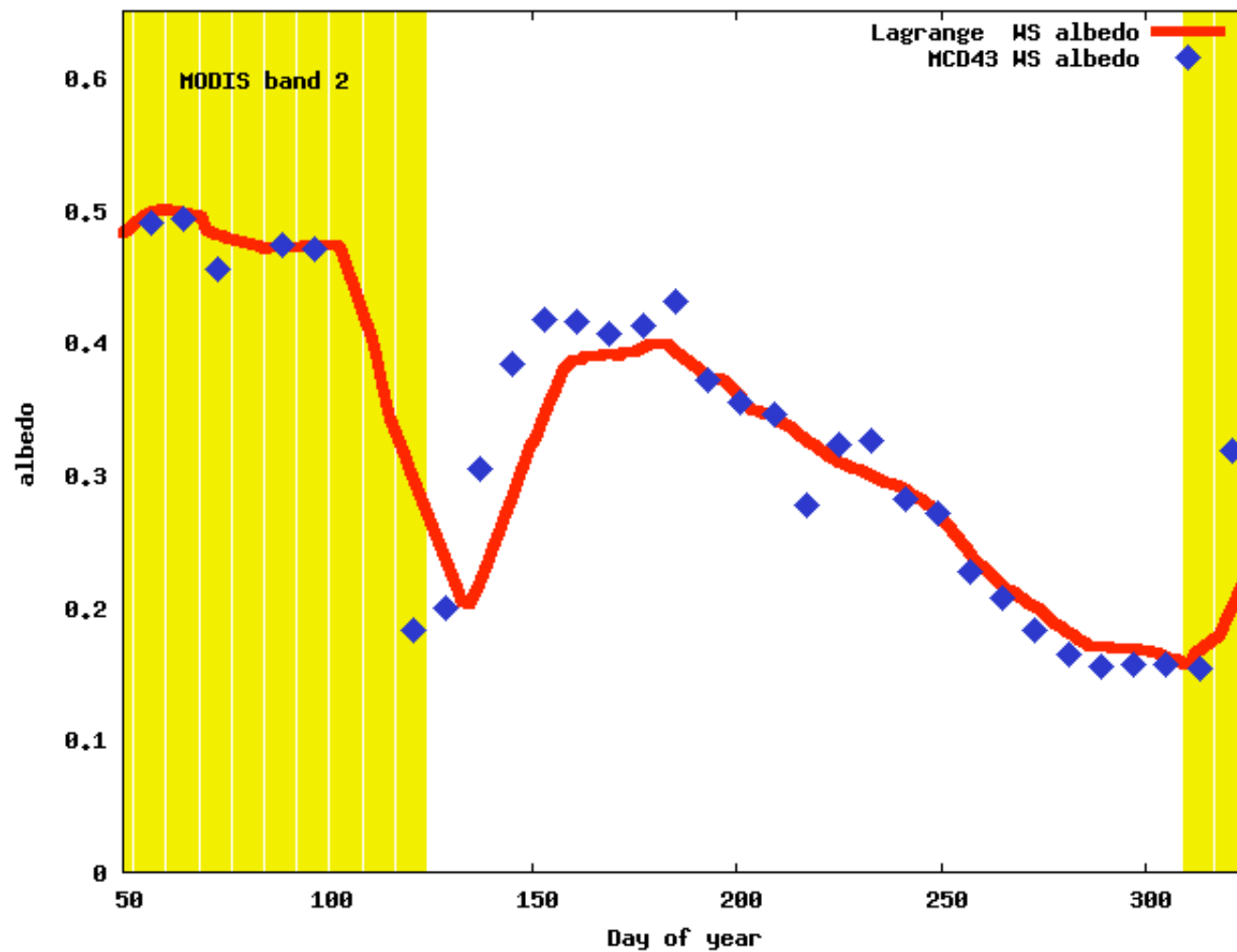


# MODIS grid



# Siberia h23v03

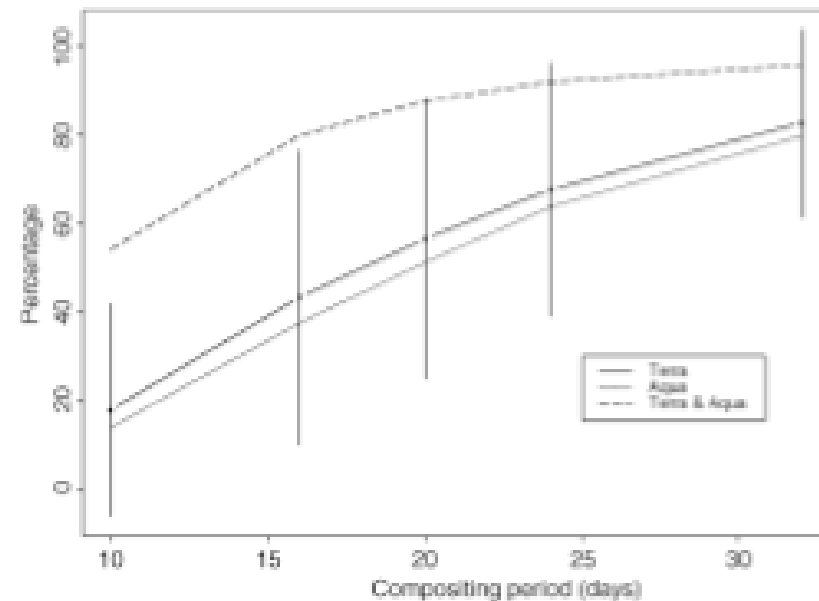
## Deciduous forest





# Model inversion

- ◆ sample and assume constant over window
  - But often clouds
    - ◆ Lack of samples
    - ◆ ill-posed
  - strategies
    - ◆ ‘backup’
      - e.g. magnitude inversion
    - ◆ Extend compositing period
    - ◆ Moving window



Global mean % of compositing periods over year  
where  $p(n_{\text{obs}} \geq 7) > 0.9$

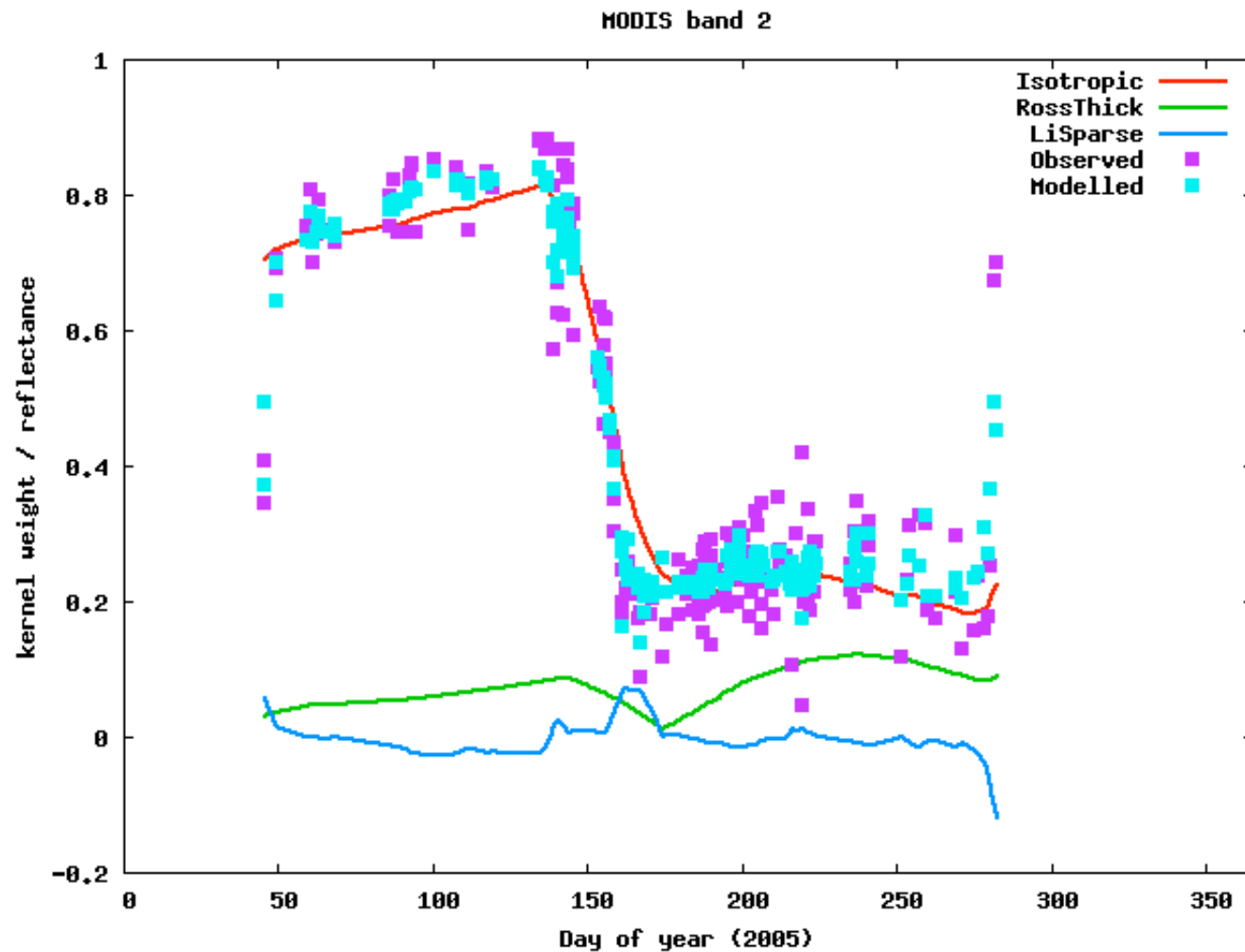
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The Global Impact of Clouds on the Production of  
MODIS Bidirectional Reflectance Model-Based  
Composites for Terrestrial Monitoring

D. P. Roy, P. Lewis, C. B. Schaaf, S. Devadiga, and L. Boschetti

# Barrow 2007



# Hmmm... barrow ...

